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DYNAMIC TRAFFIC COMPETITION IN ATM NETWORKS

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Summary: In todays heterogenous multimedia ATM networks, there are multiple network managers, multiple traffic types and every user exploits the network resources according to availability, performance and cost. Thus, there are many decision-makers competing each other, and each one tries to achieve the best for his own good.

In order a decision-maker to take the best decisions, he should have accurate information about the network state. However, in todays high speed multimedia ATM networks, the network state changes very fast and abruptly. Thus, a dynamic network model should be considered in the analysis and optimization of these high speed networks.

In this paper, we first model the dynamics of ATM networks using both the virtual call and the cell processes in a two-level dynamic model. Then we provide a dynamic game framework for the resource sharing problems in ATM networks with competing traffic. Finally, we formulate the dynamic game problem both as a Nonlinear Complementarity Problem (NCP) and as a Variational Inequality Problem (VIP).

1. Introduction

With the increase demand for distributed real-time multimedia applications such as video-on-demand, interactive TV, tele-education, tele-medicine, distributed virtual reality games etc. there is the need for high speed networks. The ATM transport and switching technique is widely viewed as the future technology for the new generation of high speed broadband communication networks. In ATM networks, fixed length packets, called cells (53 bytes), are transfered from node to node via very fast switching. Different traffic streams with wide range of characteristics are multiplexed and transfered through the same ATM network. These multiple traffic classes have widely different traffic patterns, performance constraints, modes of transport and synchronization that impose new and complex communication requirements. Although there is a lot of research on ATM networks, there are still many unanswered questions about traffic management and resource sharing.

The state of an ATM network is continually changing due to topology changes and real time traffic fluctuations. For efficient network control, the control decisions should depend on the current network state. In this paper, we consider the dynamic resource sharing problem and describe the dynamic evolution of the network state by a state space model of nonlinear difference equations. We model the problem on the path flow space and introduce two-level dynamic queueing models that describe the evolution of the average number of both virtual calls and cells in every resource for each traffic type.

The usual approach for resource sharing problems in ATM networks is to minimize a single global cost function assuming that all users cooperate in achieving it. In reality, each user has different objective and quality of service constraints from the other users and he competes with them for the shared network resources. So, each user tries to minimize his own cost and satisfy his own constraints not caring for the performance of the others. In this paper, we consider such competitive decision-makers who share the same ATM network.

Previous studies taking a game theoretic approach to computer network resource allocation problems consider the quasi-static flow control problem [1-15] and routing problem [16-19] as a *static Nash game* and the dynamic resource sharing problem [20] as a *dynamic Nash game*. In these studies, the decision-makers have equal power. When one decision-maker has more power than the other a *static Stackelberg game* approach is taken in [17, 21, 22] and a *dynamic Stackelberg game* in [17,23].

In this paper, we describe a control framework for dynamic traffic competition in ATM networks. We formulate this dynamic non-cooperative problem both as a Nonlinear Complementarity Problem and as a Variational Inequality Problem.

2. Dynamic Traffic Models

In broadband multimedia networks there are multiple decision-makers that share the same network resources. The traffic streams of these decision-makers have large variations and create bursts. In order to describe the dynamic evolution of the network state, it is not sufficient to consider only the average number of cells. In this section, we extend our work on modeling networks with virtual calls and cells [24,25] to the dynamic game formulation of resource sharing problems. We consider both the average number of virtual calls and the average number of cells per virtual call. So, we couple the virtual call and cell processes in a combined two-level model. Furthermore, we consider that the cell arrival and departure rates exhibit a drift from their average values.

We assume that virtual calls of decision-maker c arrive at the source node $[s.]$ and requires processing in any computer node that has processing capabilities. Decision-maker c should choose how many virtual calls to send to which destination node. So, let him send a fraction $\psi^c_{[sd]}(t) \geq 0$ of these virtual calls for processing to node $[.d]$ (Fig. 1). Of course, the sum of the load sharing fractions from node $[s.]$ to all of its possible destinations $[.d]$ is equal to one: $\sum_{[.d]} \psi^c_{[sd]}(t) = 1$. Define the following vectors for the load sharing fractions with their constraint sets: i) for decision-maker c: $\boldsymbol{\Psi}^c(t) \in \mathbf{LS}^c$, ii) for all other decision-makers excluding c: $\overline{\boldsymbol{\Psi}^c}(t) \in \overline{\mathbf{LS}^c}$ and iii) for all decision-makers: $\boldsymbol{\Psi}(t) \in \mathbf{LS}$.

After deciding which fraction of these virtual calls will be processed by destination node $[.d]$, the decision-maker c should choose the route from the source $[s.]$ to the destination $[.d]$. Furthermore, he may decide to reject some virtual calls outside of the network, for congestion control reasons. So, let him reject a fraction $\phi^c_{o[sd]}(t) \geq 0$ and route through path $\pi[sd]$ a fraction $\phi^c_{\pi[sd]}(t) \geq 0$ of his virtual calls for the source-destination $[sd]$ (Fig.2). Of course, all these fractions must sum to one: $\phi^c_{o[sd]}(t) + \sum_{\pi[sd]} \phi^c_{\pi[sd]}(t) = 1$. Define the following vectors for the admission and routing fractions with their constraint sets: i) for decision-maker c: $\mathbf{\Phi}^c(t) \in \mathbf{AR}^c$, ii) for all other decision-makers excluding c: $\overline{\mathbf{\Phi}^c}(t) \in \overline{\mathbf{AR}^c}$ and iii) for all decision-makers: $\mathbf{\Phi}(t) \in \mathbf{AR}$.

The real network state is a discrete-state stochastic process. However, every decision-maker cannot have instantaneous knowledge of the global state at every instant. So, even if he solves the stochastic problem, it will be difficult to implement the solution. Therefore, we propose using the deterministic approximation of this stochastic process by its expected value. We describe the network state by the average number of virtual calls as well as the average number of cells of all decision-makers at all network resources.

Let virtual calls of decision-maker c arrive at a resource with rate $\gamma^c(t)$ (Poisson distribution) and have mean duration $1/\delta^c(t)$ (general distribution). The average number of virtual calls of decision-maker c at this resource, $V^c(t)$, increases by the average number of virtual calls that arrive and decreases by the average number of virtual calls that depart during a time interval. Since, thousands of virtual call can coexist simultaneously at the resource, the virtual call departure rate is $\delta^c(t)V^c(t)$. In ATM networks, traffic fluctuates with a large variance around its average value. Therefore, we also introduce a stochastic drift for the arrival and departure rates. Thus, the following differential equation describes the dynamic evolution of the average number of virtual calls of decision-maker c at a resource:

$$\dot{V}^c(t) = \left(\gamma^c(t) - \eta^c(t) * \frac{dw^c_\eta}{dt}\right) - \left(\delta^c(t)V^c(t) - \theta^c(t) * \frac{dw^c_\theta}{dt}\right)$$

where $\eta^c(t)$ and $\theta^c(t)$ are the standard deviations of the arrival and departure rates for traffic c, and $w^c_\eta(t)$, $w^c_\theta(t)$ are Wiener processes.

Each virtual call carries cells. Let cells of decision-maker c arrive at rate $r^c(t)$ (Poisson distribution) at their corresponding virtual call. The length of every cell is $1/\mu$ (=53 Bytes for ATM networks) and the service rate at the resource is $C(t)$. Then the average number of cells of decision-maker c at this resource, $N^c(t)$, increases by the average number of cells that arrive and decreases by the average number of cells that depart during a time interval. Again, we introduce a stochastic drift for the arrival and departure rates. Thus, the following differential equation describes the dynamic evolution of the average number of cells of decision-maker c at a resource:

$$\dot{N}^c(t) = \left(r^c(t)V^c(t) - a^c(t) * \frac{dw^c_a}{dt}\right) - \left(\mu C(t)\rho^c(t) - b^c(t) * \frac{dw^c_b}{dt}\right)$$

where $\rho^c(t)$ is the instantaneous utilization for traffic c at this resource, $a^c(t)$ and $b^c(t)$ are the standard deviations of the arrival and departure rates for traffic c, and $w^c_a(t)$, $w^c_b(t)$ are Wiener processes.

Taking into account the previous ideas, we extend the dynamic models of [17,20] to the following models:

$$\dot{N}^c(t) = \left(r^c(t)V^c(t) - a^c(t) * \frac{dw_a^c}{dt}\right) - \left(\mu C(t) * \frac{2N^c(t)}{2 - \overline{x^2} * \mu^2} * \right.$$

$$\left. * \frac{1 - \overline{x^2} * \mu^2 - \sum_k N^k(t) + \sqrt{\left(1 + \sum_k N^k(t)\right)^2 - 2\sum_k N^k(t) * (2 - \overline{x^2} * \mu^2)}}{1 - \sum_k N^k(t) + \sqrt{\left(1 + \sum_k N^k(t)\right)^2 - 2\sum_k N^k(t) * (2 - \overline{x^2} * \mu^2)}} - b^c(t) * \frac{dw_b^c}{dt}\right)$$

For exponential service, general service with Processor Sharing ($P.S.$) discipline and deterministic service times, the above model gives the following dynamic models:

$$\dot{N}^c(t) = \left(r^c(t)V^c(t) - a^c(t) * \frac{dw_a^c}{dt}\right) - \left(\mu C(t) * \frac{N^c(t)}{1 + \sum_k N^k(t)} - b^c(t) * \frac{dw_b^c}{dt}\right)$$

$$\dot{N}^c(t) = \left(r^c(t)V^c(t) - a^c(t) * \frac{dw_a^c}{dt}\right) - \left(\mu C(t) * \frac{v^c * N^c(t)}{1 + \sum_k v^k * N^k(t)} - b^c(t) * \frac{dw_b^c}{dt}\right)$$

$$\begin{aligned}\dot{N}^c(t) &= \left(r^c(t)V^c(t) - a^c(t) * \frac{dw_a^c}{dt}\right) \\ &\quad - \left(\mu C(t) * 2N^c(t) * \frac{-\sum_k N^k(t) + \sqrt{1 + \left(\sum_k N^k(t)\right)^2}}{1 - \sum_k N^k(t) + \sqrt{1 + \left(\sum_k N^k(t)\right)^2}} - b^c(t) * \frac{dw_b^c}{dt}\right)\end{aligned}$$

We can rewrite the above model using the zero mean, unit variance normal random variables $\overline{\xi_a^c}(t) = \dfrac{dw_a^c(t)}{dt}$ and $\overline{\xi_b^c}(t) = \dfrac{dw_b^c(t)}{dt}$. For example, the M/D/1 dynamic model becomes:

$$\begin{aligned}\dot{N}^c(t) &= \left(r^c(t)V^c(t) - a^c(t) * \xi_a^c(t)\right) \\ &\quad - \left(\mu C(t) * 2N^c(t) * \frac{-\sum_k N^k(t) + \sqrt{1 + \left(\sum_k N^k(t)\right)^2}}{1 - \sum_k N^k(t) + \sqrt{1 + \left(\sum_k N^k(t)\right)^2}} - b^c(t) * \xi_b^c(t)\right)\end{aligned}$$

Taking all these differential equations for the average number of virtual calls and the average number of cells of all decision-makers at all network resources, we have the dynamic evolution of the network state:

$$\dot{\mathbf{X}}(t) = \mathbf{\Lambda}(t, \mathbf{X}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t)) - \mathbf{D}(t, \mathbf{X}(t))$$

where $\mathbf{X}(t) \geq 0$ is the vector network state, $\mathbf{\Lambda}(t, \mathbf{X}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t)) \geq \mathbf{0}$ is the vector arrival rate and $\mathbf{D}(t, \mathbf{X}(t)) \geq \mathbf{0}$ is the vector departure rate at time t.

Rewritting the above differential vector equation as a vector differential function, we have

$$\dot{\mathbf{X}}(t) = \mathbf{f}(t, \mathbf{X}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t))$$

Let $[\mathbf{\Phi}^c, \mathbf{\Psi}^c]$ and $[\mathbf{\Phi}, \mathbf{\Psi}]$, be the strategy of decision-maker c and of all decision-makers, respectively, during the whole duration of the problem. The decision-maker c has instantaneous at time t cost function, $g^c(t, \mathbf{N}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t))$, and total cost function during the whole duration of the problem, from the initial time t_0 to the final time t_f:

$$J^c(\mathbf{\Phi}, \mathbf{\Psi}) = \int_{t_0}^{t_f} g^c(t, \mathbf{X}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t))dt$$

For the infinite horizon problem, he has the following cost function

$$J^c(\mathbf{\Phi}, \mathbf{\Psi}) = \int_{t_0}^{\infty} e^{-kt} g^c(t, \mathbf{X}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t))dt$$

where k is a discount cost.

Examples of his cost are: i) the sum of his average cost (eg. delay) at every network resource minus his benefit (eg. throughput) from using the network, ii) the maximum penalty (eg. blocking probability) minus the minimum reward at any network resource. Here, we examine the most general case where the cost function at a resource may depend on the traffic over the whole network. Desired properties of a cost function are to be nonnegative, bounded from above, nondecreasing, continuous, differentiable and convex $\forall \mathbf{X}(t) \geq 0$.

Let also define the Hamiltonian for the decision-maker c to be:

$$H^c(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{P}^c) = g^c(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi}) + \mathbf{P}^c * \mathbf{f}(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi})$$

where $\mathbf{P}^c$: vector of costate variables associated with the state equations.

Let also the Lagrangian for the decision-maker c be:

$$\begin{aligned} L^c(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{P}^c, \mathbf{Q}^c) \; &= H^c(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{P}^c) + \sum_{[s.]} Q^c_{[s.]} * \left[1 - \sum_{[.d]} \psi^c_{[sd]}\right] \\ &+ \sum_{[sd]} Q^c_{[sd]} * \left[1 - \phi^c_{o[sd]} - \sum_{\pi[sd]} \phi^c_{\pi[sd]}\right] \end{aligned}$$

with $\phi^c_{o[sd]},\ \phi^c_{\pi[sd]},\ \psi^c_{[sd]} \geq 0 \quad \forall\ \pi[sd],\ [sd],\ c,$

where $\mathbf{Q}^c = [...\ Q^c_{[sd]}\ ...\ Q^c_{[s.]}\ ...]$: vector of the multipliers associated with the constraints of the admission, routing and load sharing fractions.

In the next sections, we consider that the multiple decision-makers compete for the limited network resources and try to use the resources of the network for their own benefit, ignoring the penalty they cause to the other decision-makers. When the decision-makers are in equilibrium, no decision-maker can improve his cost by altering his decision unilaterally [26,27,28]. We express the non-cooperative equilibrium conditions both as a Nonlinear Complementarity Problem (NCP) and as a Variational Inequality Problem (VIP). The proofs of the Theorems follow a similar approach as in [17,19] and therefore they are ommitted.

3. Nonlinear Complementarity Problem (NCP)

In this section, we formulate the dynamic non-cooperative load sharing, routing and admission control problem as a Nonlinear Complementarity Problem (NCP).

Let define the vector of the admission control, routing and load sharing fractions and of the Lagrange multipliers:

$$\mathbf{Z}(t) = \Big[...\ \phi^c_{o[sd]}(t)\ ...\ \phi^c_{\pi[sd]}\ ...\ Q^c_{[sd]}(t)\ ....\ \psi^c_{[sd]}(t)\ ...\ Q^c_{[s.]}(t)\ ...]^T$$

and the vector of the derivatives of the Lagrangian with respect to the admission control, routing and load sharing fractions as well as the Lagrange multipliers:

$$\begin{aligned}\nabla L(\mathbf{Z}(t)) \quad = \quad & \Bigg[... \left(\frac{\partial H^c}{\partial \phi^c_{o[sd]}} - Q^c_{o[sd]}(t) \right) \ ... \ \left(\frac{\partial H^c}{\partial \phi^c_{\pi[sd]}} - Q^c_{\pi[sd]}(t) \right) \ ... \\ & ... \left(1 - \phi^c_{o[sd]}(t) - \sum_{\pi[sd] \in \mathbf{\Pi}^c_{[sd]}} \phi^c_{\pi[sd]}(t) \right) \ ... \\ & ... \left(\frac{\partial H^c}{\partial \psi^c_{[sd]}} - Q^c_{[s.]}(t) \right) \ ... \ \left(1 - \sum_{[.d] \in \mathbf{D}^c_{[s.]}} \psi^c_{[sd]}(t) \right) \ ... \Bigg]\end{aligned}$$

Theorem 1:

Consider the dynamic joint load sharing, routing and admission control problem in networks with multiple competing decision-makers, with fixed initial time t_0 *and final time* t_f*. If for each decision-maker c,* g^c *is differentiable and convex in* $(\mathbf{\Phi}^c(t), \mathbf{\Psi}^c(t)) \in (\mathbf{AR}^c, \mathbf{LS}^c)$*, for each fixed value of* $(\overline{\mathbf{\Phi}^c}(t), \overline{\mathbf{\Psi}^c}(t)) \in (\overline{\mathbf{AR}^c}, \overline{\mathbf{LS}^c})$ *then* $(\mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t)) \in (\mathbf{AR}, \mathbf{LS})$ *is a Nash equilibrium if and only if it solves the following Nonlinear*

Complementarity Problem $\forall\ t \in [t_0, t_f]$:

$$\begin{aligned}
&\nabla L(\mathbf{Z}^*(t)) * \mathbf{Z}^*(t) = 0\\
&\nabla L(\mathbf{Z}^*(t)) \geq 0\\
&\mathbf{Z}^*(t) \geq 0\\
&\textit{with}\ \dot{\mathbf{X}}^*(t) = \mathbf{f}(t, \mathbf{X}^*(t), \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t))\\
&\qquad \mathbf{X}^*(t_0) = \mathbf{X}_0\\
&\qquad \dot{\mathbf{P}}^c(t) = -\nabla_{\mathbf{X}} H^c(t, \mathbf{X}^*, \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t), \mathbf{P}^c(t)) \quad \forall\ c\\
&\qquad \mathbf{P}^c(t_f) = \mathbf{0} \quad \forall\ c
\end{aligned}$$

4. Variational Inequality Problem (VIP)

In this section, we formulate the dynamic non-cooperative load sharing, routing and admission control problem as a Variational Inequality Problem (VIP).

Let define the vector of derivatives of the Hamiltonian with respect to the admission control, routing and load sharing fractions:

$$\nabla H(t, \mathbf{X}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t), \mathbf{P}(t)) \quad = \quad \left[\ldots \frac{\partial H^c}{\partial \phi^c_{o[sd]}} \ldots \sum_{\pi[sd] \in \mathbf{\Pi}^c_{[sd]}} \frac{\partial H^c}{\partial \phi^c_{\pi[sd]}} \ldots \frac{\partial H^c}{\partial \psi^c_{[sd]}} \ldots \right]$$

Theorem 2:

Consider the dynamic joint load sharing, routing and admission control problem in networks with multiple competing decision-makers, with fixed initial time t_0 and final time t_f. Let for each decision-maker c, $g^c(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi})$, $\mathbf{f}(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi})$, be continuously differentiable with respect to $(\mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi}) \in (\mathbf{R}^n, \mathbf{\Phi}, \mathbf{\Psi})$ $\forall\, t \in [t_0, t_f]$. If H^c is continuously differentiable and convex in $(\mathbf{X}, \mathbf{\Phi}^c, \mathbf{\Psi}^c) \in (\mathbf{R}^n, \mathbf{AR}^c, \mathbf{LS}^c)$, $\forall\ t \in [t_0, t_f]$, for each fixed value of $(\overline{\mathbf{\Phi}^c}(t), \overline{\mathbf{\Psi}^c}(t)) \in (\overline{\mathbf{AC}^c}, \overline{\mathbf{LS}^c})$, then $(\mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t)) \in (\mathbf{AR}, \mathbf{LS})$ is a Nash equilibrium if and only if it solves the following *Variational Inequality Problem* $\forall\ t \in [t_0, t_f]$:

$$\begin{aligned}
&\nabla H(t, \mathbf{X}^*(t), \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t), \mathbf{P}(t)) * ((\mathbf{\Phi}, \mathbf{\Psi}) - (\mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t))) \geq 0\\
&\qquad\qquad\qquad\qquad\qquad\qquad \forall\ (\mathbf{\Phi}, \mathbf{\Psi}) \in (\mathbf{AR}, \mathbf{LS})\\
&\textit{with}\ \dot{\mathbf{X}}^*(t) = \mathbf{f}(t, \mathbf{X}^*(t), \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t))\\
&\qquad \mathbf{X}^*(t_0) = \mathbf{X}_0\\
&\qquad \dot{\mathbf{P}}^c(t) = -\nabla_{\mathbf{X}} H^c(t, \mathbf{X}^*, \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t), \mathbf{P}^c(t)) \quad \forall\ c\\
&\qquad \mathbf{P}^c(t_f) = \mathbf{0} \quad \forall\ c
\end{aligned}$$

Another equivalent VIP formulation is given in the following Theorem:

Theorem 3:

Consider the dynamic joint load sharing, routing and admission control problem in networks with multiple competing decision-makers, with fixed initial time t_0 and final time t_f. Let for each decision-maker c, $g^c(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi})$, $\mathbf{f}(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi})$, be continuously *differentiable with respect to* $(\mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi}) \in (\mathbf{R}^n, \mathbf{\Phi}, \mathbf{\Psi})\ \ \forall\, t \in [t_0, t_f]$. *If* H^c *is continuously differentiable and convex in* $(\mathbf{X}, \mathbf{\Phi}^c, \mathbf{\Psi}^c) \in (\mathbf{R}^n, \mathbf{AR}^c, \mathbf{LS}^c)$, $\quad \forall\ t \in [t_0, t_f]$, *for each fixed value of* $(\overline{\mathbf{\Phi}^c}(t), \overline{\mathbf{\Psi}^c}(t)) \in (\overline{\mathbf{AR}^c}, \overline{\mathbf{LS}^c})$, *then* $(\mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t)) \in (\mathbf{AR}, \mathbf{LS})$ *is a Nash equilibrium if and only if it solves the following* <u>*Variational Inequality Problem*</u> $\forall\ t \in [t_0, t_f]$:

$$
\begin{aligned}
&\nabla L(\mathbf{Z}(t)^*) * (\mathbf{Z} - \mathbf{Z}(t)^*) \geq 0 \qquad \forall\ \mathbf{Z} \geq 0 \\
&\textit{with}\ \dot{\mathbf{X}}^*(t) = \mathbf{f}(t, \mathbf{X}^*(t), \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t)) \\
&\qquad \mathbf{X}^*(t_0) = \mathbf{X}_0 \\
&\qquad \dot{\mathbf{P}}^c(t) = -\nabla_{\mathbf{X}} H^c(t, \mathbf{X}^*, \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t), \mathbf{P}^c(t)) \quad \forall\ c \\
&\qquad \mathbf{P}^c(t_f) = \mathbf{0} \quad \forall\ c
\end{aligned}
$$

5. CONCLUSIONS

Load sharing, routing and admission control are fundamental problems in computer networks. We consider all these problems simultaneously in ATM networks with multiple decision-makers. Every decision-maker assigns his virtual calls for processing to destination processors in such a way as to optimize his own performance. At the same time, he routes them to the selected destinations through the best paths for his own performance. Every decision-maker is in competition with the other decision-makers for the shared network resources. We introduce a two-level dynamic queueing model to describe the dynamic evolution of the average number of both virtual calls and cells at every network resource. Then, we formulate the dynamic problem as a dynamic Nash game and give the non-cooperative equilibrium conditions both as a Nonlinear Complementarity Problem and as a Variational Inequality Problem.

Extensions of this work would be to solution of the dynamic problem as a stochastic optimal control problem, with state constraints, with delayed information about the network state, and with imperfect state observation.

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